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ENTIRE PATHOS EDGE SEMIENTIRE BLOCK GRAPH

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ABSTRACT

In this paper, we introduce the concept ofentire pathos edge semientire block graph of a tree E_{Pe} (T). We obtain some properties of this graph. We study the characterization of graphs whose entire pathos edge semientire block graphs are always planar, minimally nonouter planar, crossing number one. Further, we also establish the characterization for E_{Pe} (T) to be Hamiltonian and noneulerian.

KEYWORDS: Block Graph, Edge Semi Entire Graph, Inner Vertex Number, Line Graph

MATHEMATICS SUBJECT CLASSIFICATION: 05C

I. INTRODUCTION

All graphs considered here are finite, undirected without loops or multiple edges. Any undefined term or notation in this paper may be found in Harary [2].

The concept of pathos of a graph G was introduced by [1] as a collection of minimum number of edge disjoints open paths whose union is G. The path number of a graph G is the number of path of pathos. Stanton [7] and Harary [2] have calculated the path number of certain classes of graphs like trees and complete graphs.

For a graph G(p, q) if B = { u_1, u_2, u_3, · · ·, u_r; r ≥2} is a block of , then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks.

By a plane graph G we mean embedded in the plane as opposed to a planar graph. In a plane graph G let e_1=uv be an edge. We say e_1 is adjacent to the vertices u and v, which are also adjacent to each other. Also an edge e_1 is adjacent to the edge e_2 =uw. A region of G is adjacent to the vertices and edges which are on its boundary, and two regions of G are adjacent if their boundaries share a common edge.

The crossing number c(G) of G is the least number of intersection of pairs of edges in any embedding of G in the plane. Obviously G is planar if and only if c(G)=0.

The *edge degree* of an edge e = {a, b} is the sum of degrees of the end vertices a and b. Degree of a block is the number of vertices lies on a block. *BlockdegreeB*$_v$ of a vertex v is the number of blocks in which v lies. *Block path* is a path in which each edge in a path becomes a block. If two paths p_1 and $p_2$contain a common cut vertex then they are adjacent paths and the *pathdegree*P_v of a vertex v is the number of paths in which v lies. Degree of a region is the number of vertices lies on a region. The *regiondegree*R_v of a vertex v is the number of regions in which the vertex v lies. Pendant pathos is a path p_i of pathos having unit length.

The inner vertex number i(G) of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane.

A new concept of a graph valued functions called the edge semi Entire block graph $E_b(G)$ of a plane graph G was introduced by Venkanagouda in[8] and is defined as the graph whose vertex set is the union of set of edges, set of blocks and set of regions of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent, the corresponding edges lies on the blocks, the corresponding edges lies on the region and the corresponding blocks are incident to a cut vertex.

The pathos edge semientire graph $P_e(T)$ of a tree was introduced by in [9]. The pathos edge semientire graph $P_e(T)$ of a tree T is the graph whose vertex set is the union ofset of edges, regions and the set of pathos of pathos in which two vertices are adjacent ifand only if the corresponding edges of T are adjacent, edges lies on the region and edgeslies on the path of pathos. Since the system of path of pathos for a tree T is not unique, thecorresponding pathos edge semientire graph is also not unique.

The pathos edge semientire block graph of a tree T denoted by $P_{vb}(T)$ is the graph whose vertex set is the union of the vertices , regions, blocks and path of pathos of T in which two vertices are adjacent if and only if they are adjacent vertices of T or vertices lie on the blocks of T or vertices lie on the regions of T or the adjacent blocks of T. Clearly the number of regions in a tree is one. This concept was introduced by Venkanagouda in [3].

The block graph B(G) of a graph G is the graph whose vertex set is the set of blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent. This graph was studied in [2].

The path graph P(T) of a tree is the graph whose vertex set is the set of path of pathos of T in which two vertices of P(T) are adjacent if the corresponding path of pathos have a common vertex.

The following will be useful in the proof of our results.

Theorem 1[6]: If G is a (p, q) graph whose vertices have degree d_i then L(G) has q vertices andq_L edges where $q_L = -q + \frac{1}{2}\sum d_i^2$.

Theorem 2[6]: The line graph L(G) of a graph G has crossing number one if and only if G is planar and 1 or 2 holds:

1. The maximum degree D (G) is 4 and there is unique non cut vertex of degree.

2. The maximum degree D (G) is 5, every vertex of degree 4 is a cut vertex, there is a unique vertex of degree 5 and has at most 3 edges in any block.

Theorem 3[2]: A connected graph G is isomorphic to its line graph if and only if it is a cycle.

Theorem 4[6]: The line graph L(G) of a graph is planar if and only if G is planar, $\Delta \leq 4$ and if deg v = 4 for a vertex v of G, then v is a cut vertex.

Theorem 5[1]: A graph is planar if and only if it has no sub graph homeomorphic to K_5 or $K_{3,3}$.

Theorem 6[1]: A connected graph G is eulerian if and only if each vertex in G has even degree.

Theorem 7[2]: A nontrivial graph is bipartite if and only if all its cycles are even.

Theorem 8[3]: For any planar graph G, pathos edge semientire block graph $PE_b(G)$ whose vertices have degree d_i has (2q+k+1) vertices and $\frac{1}{2}\sum d_i^2+\sum q_j+\sum \frac{b_k(b_k-1)}{2}$ edges, where r the number of regions , b the number of blocks q_j the number of edges in a block b_j, b_k be the block degree of a cut vertex C_k and q_r be the number of edges region r_1.

Theorem 9 [10]: For any tree T, $P_{Eb}(T)$ has crossing number one , if and only if T is a path P_4.

Theorem 10[10]: For any tree T, $P_{Eb}(T)$ is always non-separable.

Theorem 11[10]: For any edges e_i, in a tree T with edge degree n then degree of a corresponding vertex in $T_{Pe}(T)$ is n+1.

1. Entire Pathos Edge Semi Entire Block Graph

We now define the following graph valued function.

Definition 2.1: The Entire pathos edge semientire block graph of a tree T denoted by E_{Pe} (T) is the graph whose vertex set is the union of set of edges, blocks, regions and path of pathos of T in which two vertices are adjacent if the corresponding edges of T are adjacent or corresponding blocks of T are adjacent or one corresponds to a block of T and other to the edge e of T and e lies on it, or one corresponds to a region of T and other to an edge e of T and e lies on it or one corresponds to a path of pathos of T and other to an edge e of T and e lies on it or one corresponds to the block b of T and other the path p of T and both b and p have a common edge in T. In Figure 2.1, a graph G and its total pathos edge semientire block graph are shown.

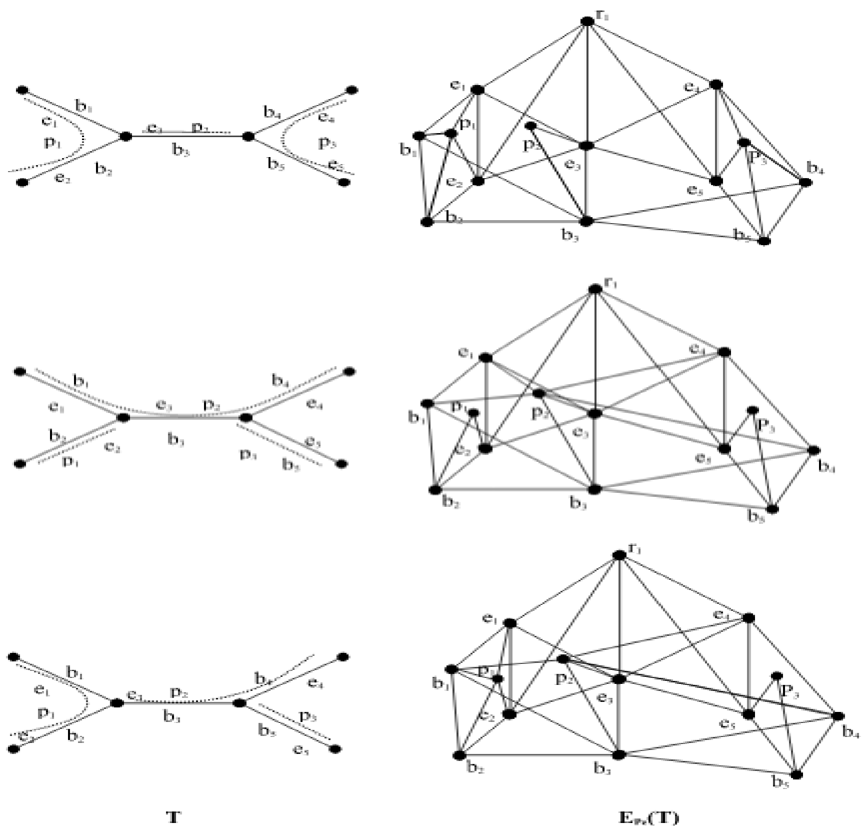

Figure 2.1

Remark 1: If a tree T is connected then $E_{Pe}(T)$ is also connected.

Remark 2: For any tree T, $P_{Eb}(T)$ is a spanning sub graph of $E_{Pe}(T)$.

Theorem 12: For any tree T, $E_{Pe}(T)$ is always nonseparable.

Proof. By Theorem 10, $P_{Eb}(T)$ is nonseparable and by Remark 2, $E_{Pe}(T)$ is also non separable.

Theorem 13: If Tis a connected graph with p vertices and q edges whose vertices have degree $d_i$and if b_i be the number of blocks to which the edge e_i belongs in T then the entire pathos edge semientire block graph$E_{Pe}(T)$ has 2q+k+1 vertices and$2q + \frac{1}{2}\sum d_i^2 + \sum q_j + \frac{1}{2}\sum b_k(b_k - 1) +$ edges, where q_j be the number of edges in each block b_j and b_k be the block degree of a cut vertex c_k.

Proof. By the definition of $E_{Pe}(T)$, the number of vertices is the union of the edges, regions, blocks and path of pathos of T. By the Remark 2 and by Theorem 8, $E_{Pe}(T)$ has 2q + k + 1 vertices. By the definition of $E_{Pe}(T)$, the number of edges in $E_{Pe}(T)$ is the sum of the edges in $P_{Eb}(T)$ and the edges of T. By Theorem 8, E $[P_{Eb}(T)] = \frac{1}{2}\sum d_i^2 + \sum q_j + \sum \frac{b_k(b_k-1)}{2}$ and the edges of pathos block graph is q. Hence E $[E_{Pe}(T)] = 2q + \frac{1}{2}\Sigma {d_i}^2 + \Sigma q_j + \frac{1}{2}\Sigma b_k(b_{k-1})$.

Theorem 14: For any tree T, $E_{Pe}(T)$ is planar if and only if T is $K_{1,n}$, $n \leq 3$.

Proof. Suppose $E_{Pe}(T)$ is planar. Assume that T is $k_{1,n}$ for $n \geq 4$.For the sake of simplicity, we take n = 4. By the definition of L(T), L $(K_{1,4}) = K_4$. Since all the edges of T lies on one region then in $E_{Pe}(T)$, the corresponding region vertex r_1 is adjacent for all vertices of K_4 to form a complete graph K_5. Hence $E_{Pe}(T)$ contain K_5 as induced sub graph, which is nonplanar, a contradiction.

Conversely, suppose $T=K_{1,n}$, for $n \leq 3$.Let $T = K_{1,3}$ then $L(K_{1,3}) = K_3$. Since T is a tree and all the edges lies on only one region r_1. In a tree T each edge is a block then by the definition of $E_{Pe}(T)$, each vertex e_1' is adjacent to the block vertices b_i as well as the region vertex r_1. Further the pathos vertices p_i is adjacent to the vertices${e_i}'\&{e_j}'$ which are corresponds to the edges lies on path of pathos of T.Lastly, each block is an edge and is adjacent to the pathos vertices to form a planar graph. Hence $E_{Pe}(T)$ is planar.

Theorem 15: For any tree T, $E_{Pe(T)}$ is minimally nonouterplanar if and only if $\Delta(T) \leq 2$ and T has a unique vertex of degree 2.

Proof. Suppose $E_{Pe}(T)$ is minimally nonouterplanar assume that $\Delta(T) < 2$. By the Theorem 14, $E_{Pe}(T)$ is outer planar, a contradiction. Thus $\Delta(T) = 2$.

Assume that there exist two vertices of degree 2 in T then by Theorem 9, $P_{Ee}(T)$ has crossing number one which is non-planar, a contradiction. Hence T has exactly one vertex of degree 2

Conversely, suppose every vertex of T has ≤ 2 and has a unique vertex of degree 2, there T is P_3. By the definition of timegraph $L(P_3) = P_2$. Since all edges of T lies on only one region & it contain only one path. In $E_{Pe}(T)$ r_1 is adjacent to $e_i\&e_j$ and $e_i\&e_j$ adjacent to the block b_i, b_j respectively. Also b_i and bj are adjacent to form cycle e_i, e_j, b_j, b_ie_i ,r_i is adjacent to $e_i\&e_j$. Further P_i is adjacent to e_i, e_j, b_i,b_j , clearly p_i is the inner vertex number. Hence I $[E_{Pe}(T)] = 1$.

Theorem 16: For any tree T, $E_{Pe}(T)$ has crossing number one if and only if T is a path P_4 or $K_{1,3}(P_1)$.

Proof. Suppose $E_{Pe}(T)$ has crossing number one, then $E_{Pe}(T)$ is non-planar. By the Theorem 14 we have T = $K_{1,n}$, n≥ 4 or T= P_n n≥ 4.

We now consider the following cases.

Case 1: Assume that T = $K_{1,n}$ for n = 4. By the definition of line graph L ($K_{1,4}$) = K_4. In a tree all the edges lies on only one region r_1, in $T_{Ep}(T)$, r_1 is adjacent to all vertices of K_4, which form K_5.Further each edge is a block in T and all four blocks b_1, b_2, b_3& b_4 are adjacent to each other to form a complete graph K_4={ b_1, b_2, b_3, b_4}. In $E_{Pe}(T)$, the inner vertex say b_2 is adjacent to the corresponding vertices $e_i{}'$ which form one more crossing number. Hence $E_{Pe}(T)$ has crossing number at least two, a contradiction.

Case 2: Assume that T = P_n, for n = 5 clearly L(P_5)=P_4. In $E_{Pe}(T)$.the region vertex r_1 is adjacent to all vertices $e_1{}', e_2{}', e_3{}', e_4{}'$ which corresponds to the edges e_1, e_2, e_3, e_4 of T and each $e_1{}'$ is adjacent to b_i. Since all edges lies on only one path we join the vertices $e_i{}'$ to the pathos vertices P_i. Clearly its crossing number is at least two, a contradiction.

Case 3: Assume that T be a graph $\sigma = K_{1,3}\ (P_2)$. By Theorem 15,$E_{Pe}(T)$ is nonplanar. The graph σ contains two path of pathos and their corresponding to two pathos vertices p_1 and $p_2E_{Pe}(T)$. These two vertices lies in the interior region of $E_{Pe}(T)$. Also they have joined by the edge and gives crossing number at least two, a contradiction.

Conversely, suppose T is $K_{1,3}(P_1)$. By Theorem 15,$E_{Pe}(T)$ is planar. $K_{1,3}$ (P_1) contains two path of pathos p_1&p_2 such that p_1 lies in the interior region and p_2 lies in the exterior region. In $E_{Pe}(T)$, two vertices joined by the edges e_1, e_2, for p_1 and e_3, e_4 for p_2, gives crossing number one. Hence $E_{Pe}(T)$ has crossing number one.Also T is P_4, then by Theorem 15$E_{Pe}(P_4)$ is nonplanar. In a path P_4, there is only one pathosvertex p_1 which is adjacent to the vertices $e_1{}', e_2{}', e_3{}'$ which corresponds to the edges e_1, e_2, e_3 of P_4. Also e_1, e_2, e_3, b_1, b_2, b_3 form $2C_4$ cycles. Since P_4 contains only one region r_1 which is adjacent to all $e_i{}'\ \forall\ i$ and gives crossing number one. Hence $E_{Pe}(P_4)$ has crossing number one.

Theorem 17: For any tree T, $E_{Pe}(T)$ is always noneulerian.

Proof. Let T be a non-trivial tree we consider the following cases.

Case 1: Suppose T be a path P_n. If n = 3, both edges having edge degree odd, by Theorem 11, both vertices have even degreein $E_{Pe}(T)$. ut the block vertices b_i is adjacent to b_j, e_i and p_1 to get odd degree. Hence $E_{Pe}(P_3)$is noneulerian. If n ≥ 3, then the internal edges having edge degree even. By Theorem 11, the corresponding vertices in $E_{Pe}(T)$ have odd degree. Then $E_{Pe}(T)$ is noneulerian.

Case 2: Suppose T be a star $K_{1,n}$. If n is odd then each edge having edge degree even. In $E_{Pe}(T)$, the corresponding vertices having degree odd, which is noneulerian. If n is even then each edge having edge degree odd. In $E_{Pe}(T)$ the corresponding vertices $e_i{}'$ having even degree. Further each block vertices is adjacent to all the each remaining n-1 block vertices to form complete graph K_n. Also each block b_i is adjacent to the vertices $e_i{}'$ corresponding to $e_i \in b_i$ and P_i is adjacent to b_i gives a vertex $b_i$having odd degree. Hence $E_{Pe}(T)$ is non eulerian.

Case 3: Suppose T be any tree.By case 1, 2, $E_{Pe}(T)$ is noneulerian. Hence $E_{Pe}(T)$ is always noneulerian.

Theorem 18: For any tree T, $E_{Pe}(T)$ is always Hamiltonian .

Proof. We consider the following cases.

Case 1: Suppose T is a path with [e_1, e_2. . . . . .e_n] ∈ E(T) and $b_1 = e_1$, $b_2 = e_2$, ...$b_n = e_n$ be the blocks of T. T has exactly one path of pathos and only one region r_1. Now the vertex set of $E_{Pe}(T)$ V [$E_{Pe}(T)$] = { $e_1{}', e_2{}', \ldots\ldots e_n{}'$} U [b_1, b_2, ...b_n} U P_1 U r_1 then by the definition of $E_{Pe}(T)$, the block vertices the region vertices and the pathos vertices are adjacent to all $e_1{}', e_2{}', \ldots\ldots e_n{}'$ as shown in the figure 2.3. Clearly in $E_{Pe}(T)$ the Hamiltonian cycle r_1, $e_1{}'b_1b_2$ . . . . . b_n $e_n{}'P_1$ e'_{n-1} e'_{n-2} . . . . . .$e_2{}'$ r_1 exists. Hence $E_{Pe}(T)$ is Hamiltonian

Case 2: Suppose T is not a path then T has at least one vertex with degree at least 3. Assume that T has exactly on one vertex V such that degree > 2. Now we consider the following subcases of case 2.

Sub Case 2.1: Assume that T = $K_{1,n}$, n>2 and is odd then the number of paths of pathos are $\frac{n+1}{2}$. Let V [$E_{Pe}(T)$] = { $e_1{}', e_2{}', \ldots\ldots e_n{}'$ b_1, b_2, ...b_n} U r_1 U { P_1, $P_2 P_{\frac{n+1}{2}}$ } then there exist a cycle containing the vertices of $E_{Pe}(T)$ as $r_1e_1{}'P_1b_1b_2$. . . . .b_np_2 $e_3{}'$ . . . . .$P_{\frac{n+1}{2}}$ $e_2{}'$ r_1 and is a Hamiltonian cycle. Hence $E_{Pe}(T)$ is Hamiltonian.

Sub Case 2.2: Assume that T = $K_{1,n}$ n>2 and is even then the number of path of pathos are n/2. Let V [($E_{Pe}(T)$] = { $e_1{}^I, e_2{}^I, \ldots\ldots e_n{}^I$ b_1, b_2, ...b_n}U r_1 U [P_1, P_2. . . . . $P_{\frac{n}{2}}$]. By the definition of P $E_{Pe}(T)$, there exists a cycle containing the vertices of $E_{Pe}(T)$ as $r_1e_1p_1b_1b_2$ . . . . . .$b_nP_2e_4$ . . . .$P_{\frac{n}{2}}$ e_3 e_2 r_1 and is a Hamiltonian cycle. Hence $E_{Pe}(T)$ is Hamiltonian.

Case 3: Suppose T is neither a path nor a star then T contains at least two vertices of degree greater than 2. Let V [$E_{Pe}(T)$] = {$e_1{}'$ $e_2{}'$. . . . . . .$e_n{}'b_1$ b_2. . . . . .b_n} U { P_1 P_2 . . . . .P_k} U r_1. By the definition of$E_{Pe}(T)$ there exist a cycle C containing all the vertices of $E_{Pe}(T)$ as r_1, $e_1{}'$, b_1, b_2, b_n, P_1, $e_3{}'$ $b_3b_4e_4{}'p_2$ . . . .$e_{n-1}{}'$ $b_{n-1}p_ke_n{}'$ r_1. Hence $E_{Pe}(T)$ is a Hamiltonian cycle. Clearly $E_{Pe}(T)$ is a Hamiltonian graph.

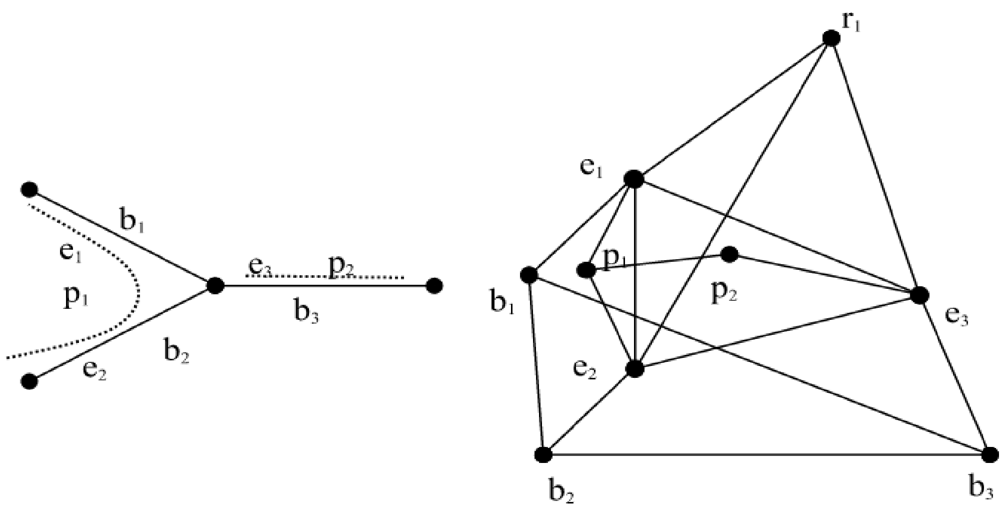

Figure 2.2

CONCLUSIONS

In this paper, we introduced the concept of the entire pathos edge semientire block graph of a tree. We characterized the graphs whose entire pathos edge semientire block graphsare planar, noneulerian, Hamiltonian and crossing number one.

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